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# Integrating educational quality and educational equality into a model of mathematics performance

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ABSTRACT

Using data from TIMSS 2015, this study investigated determinants of inequality between classrooms in mathematics performance in Sweden. Applying multiple-group confirmatory factor analysis and measurement invariance frameworks to identify latent constructs with which to build a two-level structural equation model, this study integrated teacher certification, teacher preparedness and school emphasis on academic success into a model of inequality of outcomes and opportunities. The study found evidence that more socioeconomically advantaged classes had better prepared mathematics teachers. School culture towards academic achievement was not associated with mathematics achievement. Finally, the analyses indicated that substantial inequalities exist for students taught by specialist and non-specialist teachers.

## 1. Introduction

School choice is a key feature of the contemporary Swedish school system for both teachers and students (e.g. Lundahl, 2002). Teacher recruitment is the responsibility of schools, and teachers are free to apply for positions aligning with their specialities in any locale across Sweden. In the recruitment of teachers, school principals must compete for qualified teachers (Skolverket, 2018b). With the aim of increasing student performance through enhanced competition (Lundahl, Arreman, Holm, & Lundström, 2013), the Swedish school system has seen a number of reforms since the early 1990 s. These liberalizations of the school system which include the decentralisation of control, the introduction of state-financed private schools, the right of parents to apply to send their children to their choice of school, and an increasingly marketized system, can be viewed as part of the new public management approach that has been increasingly introduced in Sweden (e.g. Björklund, Clark, Edin, Fredriksson, & Krueger, 2005; Lundahl, 2002).

Following these reforms, equality in Swedish schools has decreased (Lundahl et al., 2013), and it has been recognized that the Swedish school system has become more segregated in terms of outcomes in the period following the introduction of the free school choice policy (Myrberg & Rosén, 2006; Skolverket, 2009). The use of school choice policies is socially segregated (Teske & Schneider, 2001), with evidence emerging that the determinant of between-school segregation in Sweden has shifted from residential segregation based on the proximity-to-school principle to social background, with levels of segregation in Sweden now exceeding those of its previously residential segregated model (Böhlmark, Holmlund, & Lindahl, 2016). Swedish schools have become more segregated in terms of student achievement (Yang Hansen & Gustafsson, 2016), particularly in large metropolitan areas. Such an inequality in the distribution of achievement suggests that different school conditions and opportunities to learn are being afforded to students in different groups.

Despite the increasing achievement gap, Swedish mathematics performance in international large-scale assessments (ILSAs) appears to be undergoing something of a renaissance. After experiencing *PISA shock* in 2012 with tumbling performance levels, the achievement of young Swedes in mathematics has rebounded somewhat, with the 2018 cycle of PISA witnessing a comparable average achievement score in mathematics to the 2006 cycle (Sollerman & Winnberg, 2019). Similarly, results from the 2015 cycle of the Trends in International Mathematics and Science Study (TIMSS) indicate a nascent Swedish recovery in achievement after over a decade of decline in student performance (Skolverket, 2016).

While a body of research has established that the existence of school choice for students impacts achievement gaps between schools (e.g.

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Yang Hansen & Gustafsson, 2016), the mechanism behind this achievement gap is unclear. In the present study, we propose to investigate whether an unequal distribution of teaching preparedness in Swedish mathematics classrooms is associated with an unequal distribution of mathematics achievement. For this purpose, we will use data from eighth grade students in the 2015 cycle of TIMSS.

## 2. Previous research

Previous research has suggested that schools can foster both quality education and educational equity, with Kyriakides and Creemers (2011) noting that "schools that are among the most effective in terms of equity do not risk being among the least effective in terms of the quality dimension" (p. 248). As such, we consider educational quality and educational equality in the context of mathematics achievement using the framework of The Dynamic Model of Educational Effectiveness (Creemers & Kyriakides, 2008). In doing so, we first introduce our factors of interest and their relations to student outcomes, before considering them within the theoretical framework.

### *2.1. Educational quality*

Teachers are the most expensive learning tool available to students, and teacher quality has long been recognised in educational quality and practice as being fundamental to the achievement of students (e.g. Darling-Hammond, 2000). At the same time, however, there has been a lack of clear consensus among researchers as to which characteristics best indicate teacher quality. In Goe's (2007) framework of teacher quality, facets of teacher quality weree grouped into three areas, inputs (i.e. teacher qualification and teacher characteristics), processes (i.e. teacher practices), and outcomes (i.e. teacher effectiveness). When multiple indicators of teacher quality (certification, specialisation, and years of experience) were considered in concert, access to highly qualified mathematics teachers was shown to vary between different social groups, with high-SES students more likely to have qualified teachers than their low-SES peers. However, Sweden has demonstrated a small but compensatory distribution of qualified mathematics teachers with 3.7% more low-SES students taught by highly qualified teachers than their high-SES peers (Akiba, LeTendre, & Scribner, 2007). The aspects of Goe's teacher quality framework we explore in this study are teacher certification and teacher preparedness.

Teacher completion of degree-level coursework in mathematics has been shown to have a positive impact on student learning in mathematics, particularly for older students (Rice, 2003; Wayne & Youngs, 2003). It has long been established that students with specialist teachers fully-certified in their subject area outperform those with less qualified instructors (e.g. Darling-Hammond, 2000; Hawk, Coble, & Swanson, 1985). Socioeconomic differences in access to high quality mathematics teachers, and teachers with a major in mathematics are associated with a larger achievement gap in mathematics at the eighth grade level (Akiba et al., 2007).

Teaching is a graduate profession in Sweden. Teachers are required to have a bachelors level education in teaching, which must be verified by the National Agency for Education in order to obtain a permanent contract. In addition, teacher verification is tied to the age-group a teacher is trained to teach and subject specialism (SFS, 2011:326). In 2015, our year of interest, 94% of employed mathematics teachers across all compulsory school grades held a degree in mathematics education, and 81% of teachers in grades 7–9 (i.e. students between 13 and 16 years old) were qualified to teach this age group. Approximately 85% of mathematics teachers within grades 7–9 were licenced to teach both mathematics and this age group (Skolverket, 2015).

One of the teacher characteristics which has proven instrumental in their effectiveness is preparedness to teach their subject. Preparedness to teach dimensions within subjects has been shown to have a positive association with students' mathematics achievement (Caceres, 2009). Additionally, Yetkiner Özel and Özel (2013) found that in Turkish schools there was a discrepancy in the distribution of highly prepared teachers among different social groups. In studies using TIMSS data, teachers have reported generally feeling somewhat prepared to teacher mathematics (Burroughs et al., 2019). The teacher self-reported preparedness to teach in TIMSS has been observed to correlate with student SES (Luschei & Chudgar, 2011). However, evidence that preparedness to teach subject matter within mathematics has a direct relationship with student achievement has thus far been found to be either non-significant (Gustafsson & Nilsen, 2016) or rare (Luschei & Chudgar, 2011). Nevertheless, preparedness has been associated with instructional quality (Blomeke, Olsen, & Suhl, 2016). These earlier contributions to the literature indicate an inequality in access to prepared teachers.

Teachers operate with the broader environment of the school. An essential function of schools is to create an ethos or shared culture among members of the school community (i.e. students, parents, teachers, administration). One of the collective beliefs that a school can build its culture on concerns academic success. Such a shared belief among multiple actors within the school has been repeatedly shown to be associated with high student achievement (e.g. Bryk & Schneider, 2003; Hoy, Tarter, & Woolfolk Hoy, 2006; Kythreotis, Pashiardis, & Kyriakides, 2010; Martin, Foy, Mullis, & O'Dwyer, 2013).

Shared beliefs around academic success have been conceptualised in a variety of ways, including as collective efficacy (Hoy et al., 2006), and academic pressure or emphasis (e.g. Hoy et al., 2006; Kythreotis et al., 2010; McGuigan & Hoy, 2006). School emphasis on academic success (SEAS) is a conceptualisation of group success belief proposed by Martin et al. (2013). This conceptualisation of SEAS is drawn from teacher and principal responses to questionnaire items indicating an environment supportive of academic success. In an examination of effective schools in mathematics, literacy, and science among fourth graders, Martin et al. (2013) noted that there was a moderate correlation of about $r = 0.35$ between achievement and SEAS, making it one of the strongest school level correlates of student achievement. A strength of the SEAS construct put forward by Martin et al. (2013) is that it has been shown to be applicable across multiple national contexts with high construct validity.

The SEAS construct indicates the perceived collective beliefs and behaviours around academic success among teachers, parents, students, and the school. Nilsen and Gustafsson (2014) utilised the SEAS construct measured at the teacher level in TIMSS, and found that SEAS strongly predicted science achievement across multiple cycles of TIMSS in Norway. However, once SES was accounted for in the model, SEAS became non-significant.

Previous research has shown that there is a stronger association between parental expectations of achievement and child achievement for higher SES families (Pinquart & Ebeling, 2020). Further, teachers have higher expectations of more advantaged students (e.g. Harvey & Slatin, 1975). Given the established correlation between SES and expectations among the various stakeholders that comprise the SEAS measure, it is reasonable to assume a correlation between SES and SEAS, with schools serving more advantaged students having a stronger collective stakeholder belief in academic success.

### *2.2. Educational equality*

Socioeconomic inequality in education is a key concern of contemporary educational research and debate (i.e. Jerrim, Volante, Klinger, & Schnepf, 2019), and is perhaps the most investigated determinate of outcomes in the educational sciences (Strietholt et al., 2019). Students from more advantaged socioeconomic backgrounds have continually been demonstrated to dominate the upper end of the achievement spectrum (e.g. Mullis, Martin, Foy, & Hooper, 2016; OECD, 2016), and in many countries the relative advantage or disadvantage that a child is born into is a strong determinant of their life chances (Coleman, 1988; Sirin, 2005). Socioeconomic inequality in student achievement has been

**Table 1**
Descriptive Statistics.

| | Whole Sample | | Non-specialist Teachers | | Specialist teachers | |
|---|---|---|---|---|---|---|
| | Mean | Std. Deviation | Mean | Std. Deviation | Mean | Std. Deviation |
| Student Level | | | | | | |
| Mathematics PV1† | 5.019 | .709 | 4.943 | .719 | 5.083 | .694 |
| Mathematics PV2† | 5.019 | .725 | 4.942 | .734 | 5.084 | .711 |
| Mathematics PV3† | 5.022 | .716 | 4.952 | .719 | 5.081 | .709 |
| Mathematics PV4† | 5.027 | .729 | 4.943 | .741 | 5.099 | .712 |
| Mathematics PV5† | 5.026 | .725 | 4.950 | .723 | 5.090 | .720 |
| Books* | .000 | 1.000 | -0.023 | .999 | .019 | 1.000 |
| Parental Education* | .000 | 1.000 | -0.046 | 1.006 | .041 | .993 |
| Home Possessions* | .000 | 1.000 | -0.030 | .983 | .025 | 1.014 |
| Teacher Level | | | | | | |
| L2 Books* | .000 | 1.000 | -0.054 | .971 | .045 | 1.022 |
| L2 Parental Education* | .000 | 1.000 | -0.092 | 1.019 | .077 | .978 |
| L2 Home Possessions* | .000 | 1.000 | -0.078 | 1.005 | .065 | .992 |
| CC: Number* | .000 | 1.000 | -0.124 | .791 | .103 | 1.134 |
| CC: Algebra* | .000 | 1.000 | -0.100 | 1.002 | .083 | .991 |
| CC: Geometry* | .000 | 1.000 | -0.019 | 1.044 | .016 | .962 |
| CC: Data* | .000 | 1.000 | -0.125 | 1.014 | .102 | .977 |
| PT: Number* | .000 | 1.000 | -0.103 | 1.099 | .088 | .898 |
| PT: Algebra* | .000 | 1.000 | -0.179 | 1.237 | .150 | .712 |
| PT: Geometry* | .000 | 1.000 | -0.099 | 1.142 | .085 | .850 |
| PT: Data* | .000 | 1.000 | -0.158 | 1.169 | .128 | .816 |
| SEAS: Teacher* | .000 | 1.000 | -0.152 | 1.029 | .126 | .958 |
| SEAS: Parents* | .000 | 1.000 | -0.166 | .925 | .138 | 1.039 |
| SEAS: Students* | .000 | 1.000 | -0.029 | .975 | .024 | 1.020 |
| SEAS: School* | .000 | 1.000 | -0.137 | .978 | .112 | 1.004 |

NB. †variable is divided by 100, *standardized variable

observed across multiple cycles of TIMSS and is evident in almost all national contexts (i.e. Rolfe, Strietholt, & Yang Hansen, 2021). Across economically developed nations, socioeconomic status accounts for 14% of achievement variation in mathematics (OECD, 2019). Sweden, despite its strong social-democratic tradition and lower than average levels of income inequality, has been no exception to this trend, with socioeconomic status explaining 13% of student variation in mathematics performance, and similar achievement gaps being observed in science and literacy (OECD, 2019).

A second area of socioeconomic inequality we consider is that of opportunities. The construct used to indicate opportunities, opportunity to learn is at its' most crude, the notion that students will not successfully test on topics which they have not been taught. Although opportunity to learn has been measured in multiple ways (see Martinez Fernandez, 2005), the conceptualisation we have used in this study is that of content coverage within the subdomains of the curriculum. This approach is well established in studies of opportunity to learn using ILSA data (e.g. Schmidt, Burroughs, Zoido, & Houang, 2015). A frequent finding of international studies utilising content coverage as opportunity to learn is that it significantly predicts achievement (e.g. Luyten, 2017; Rolfe et al., 2021; Schmidt et al., 2015) and is unequally distributed among students (Schmidt et al., 2015) and classes with differing levels of socioeconomic status (Rolfe et al., 2021).

The concept of opportunity to learn can be interpreted as an embodied teacher practice in terms of delivering the curriculum, with Wang (1998) noting, "measures of students' opportunity to learn (OTL) are essential not only for interpreting students' test results, but also for evaluating the quality of their educational environments" (p. 137). The items used to indicate opportunity to learn in TIMSS data ask when content was taught to students. The response options given to teachers ("mostly taught this year", "mostly taught before this year", and "not yet taught or just introduced", see Foy, 2017b, p. 256) indicate whether students are ahead or behind in mastering the content expected of students in the eighth grade. In the Swedish school system, where the compulsory school curriculum is expressed not in grade specific knowledge, but three-year blocks of learning (Skolverket, 2018a), the integration of content coverage items into a model of educational quality and equality might also be indicative of an approach to curriculum planning and teacher behaviour at the classroom level overall. Indeed, unequal distribution of content coverage has been observed in Sweden at various time points in TIMSS data (e.g. Rolfe et al., 2021). Classes of socioeconomically advantaged students received greater content coverage of the topics assessed in TIMSS in the 2003 and 2015 cycles of the assessment (Rolfe et al., 2021), indicating that the relationship between a class' socioeconomic makeup and the educational opportunities offered is ripe for further contextualisation.

### 2.3. *Dynamic model of educational effectiveness*

The factors affecting student outcomes highlighted in the review of the literature – teacher certification and preparedness, school emphasis on academic success, and educational equality – are held to relate to one another within the framework of Creemers and Kyriakides' (2008) Dynamic Model of Educational Effectiveness. This model is centred around teaching and learning, and integrates multiple effectiveness factors which a measureable in multiple ways across the various levels of the model (e.g. student, class, and school). The model "provides a better picture of what makes teachers and schools effective but also helps us develop more specific strategies for improving educational practice" (Creemers & Kyriakides, 2008, p. 83). In this analysis, educational equality is established through student reports of socioeconomic status, providing the student level input to the Dynamic Model of Educational Effectiveness. The teacher level is represented by content coverage, teacher certification and teacher preparedness, while school emphasis on academic success represents the school level (although it is measured on the teacher level).

While the inclusion of factors representing the various levels of actors in the enterprise of education is foundational to the Dynamic Model of Educational Effectiveness, the model is particularly suitable for analysing the effects of these factors within one level, such as the classroom (see Creemers & Kyriakides, 2008). As education is by necessity a collective enterprise, with students clustered in classes and classes in schools, we consider the relations of content coverage, teacher certification, teacher preparedness, and school emphasis on academic success to collective socioeconomic status and mathematics outcomes within classrooms. Additionally, the Dynamic Model of Educational Effectiveness assumes that teachers are essential engines of change within schools, and posits that it is a framework for developing evidence-based recommendations for policy change (Creemers & Kyriakides, 2008), goals which are sympathetic to nationally-bounded analyses of international assessment data.

## 3. Study aims

The purpose of the present study is to examine how educational quality factors can be integrated into a model of educational equality in Swedish mathematics performance. The study uses representative data for Swedish eighth graders and their teachers from TIMSS 2015 to explore the themes of educational quality and equality at the classroom level by firstly identifying socioeconomic inequalities in access to highly prepared teachers and the relations between SEAS and achievement across the sample as a whole. Latterly the study investigates teacher specialisation, and whether differing patterns of socioeconomic inequalities emerge between specialist and non-specialist taught groups. As such, this study seeks to answer the following questions:

(1) Is there socioeconomic inequality in access to well-prepared mathematics teachers in Sweden?
(2) Is SEAS associated with mathematics achievement in Sweden?
(3) Do the classes of teachers with different specialties experience differing patterns of inequalities when examining the relations between educational quality and educational equality factors and mathematics achievement?

## 4. Methods

### *4.1. Data source and participants*

This study drew on the International Association for the Evaluation of Educational Achievement's (IEA) TIMSS for 2015, and used data pertaining to Swedish grade 8 students. TIMSS uses a two stage stratified sampling design, which samples firstly schools and secondly one or more classes within the selected schools (Martin, Mullis, & Hooper, 2016). As such, the sample used in this analysis is held to be representative of eighth graders and their teachers in Sweden.

### *4.2. Variables*

#### *4.2.1. Achievement*

Mathematics achievement was the achievement variable used in this analysis. The IEA standardizes TIMSS achievement scores to have a mean of 500 with a SD of 100, based on the countries that participated in 1995. In preparing our data, we divided these scores by 100, so that a score difference of 1 corresponds with one international SD, for ease of interpretation. Descriptive statistics for the achievement scores are displayed in Table 1.

#### *4.2.2. Teacher specialism*

Specialist teachers were defined as teachers with university level education in both mathematics and mathematics education. The cohort was split into the specialist and non-specialist groups using the derived variable "Teachers Majored in Math and Math Ed" provided in the TIMSS dataset (Foy, 2017c). This variable compiled teacher responses to multiple questionnaire items about their level of education and the major(s) studied. Teachers who were coded as "Major in Math and Math Ed" were thus assigned to the Specialist group, and all others were labelled non-specialist. In the 2015 TIMSS dataset, Sweden had an above average proportion of students with dual-qualified mathematics teachers (Mullis et al., 2016). The dataset included 3888 students and 190 teachers. When the cohort was separated into groups with mathematics specialist and non-mathematics specialist teachers, 2114 students were taught by 99 specialists, and 1774 students were taught by 91 non-specialists.

#### *4.2.3. Socioeconomic status*

The first of the independent variables was SES. We conceptualised SES as being indicated by three items, the number of books in the home, the highest level of parental education, and the possession of certain items in the home. The use of three items to indicate SES was congruent with established theories of the measurement of the concept (e.g. Sirin, 2005). The number of books in the home was measured in a five point scale, from 0 for "none to 10 books at home" to 4 representing "more than 200 books at home". The highest level of parental education was measured in a five point scale (0 ="no education or less than nine years of compulsory school", 1 ="lower secondary school"; 2 ="upper secondary school"; 3 ="post-secondary but not university"; 4 ="university studies"). The final item used to indicate SES was home possessions. This item was the sum of student responses to 10 items ('a computer of your own', 'a computer at home', 'a desk', 'your own room', 'internet', your own mobile phone', 'a gaming system', 'a globe', 'a piano', and 'any other instrument'), with higher values indicating possession of more of the items. These individual level variables were aggregated to class level and used to indicate the socioeconomic makeup of the students within a teacher's class. All SES indicators were standardised to a mean of 0 and a SD of 1 to ease comparability. Descriptive statistics for SES variables are displayed in Table 1.

### *4.3. Content coverage*

In this study we used content coverage to conceptualise different levels of opportunity to learn between Swedish mathematics classrooms. This approach was rooted in prior studies of opportunity to learn in ILSAs (e.g. Luyten, 2017; Rolfe et al., 2021; Schmidt et al., 2012; Schmidt et al., 2015; Schmidt & McKnight, 2012). The approach was particularly suited to analysis of TIMSS data as opportunity to learn has been indicated by teacher reports of content coverage since the earliest IEA studies (e.g. Husén, 1967; McDonnell, 1995). Teachers were questioned on whether they had taught topics within the four subdomains of mathematics (number, algebra, geometry, and data and chance) and when this content was taught to students, with the response options "mostly taught this year", "mostly taught before this year", and "not yet taught or just introduced" (Foy, 2017b, p. 256). The items were recoded so that responses of "Not yet taught or just introduced"= 0, "Mostly taught this year"= 1, and "Mostly taught before this year"= 2. Means of the items within the subdomains were taken to create the new variables: CC: Number (5 items), CC: Algebra (6 items), CC: Geometry (6 items), and CC: Data & Chance (3 items) (for a full list of the items within each parcel, see Appendix A). Finally, each of these four new variable parcels were standardized to a mean of 0 and a SD of 1. Descriptive statistics are shown in Table 1.

The operationalisation of content coverage using three teacher response options was enacted to account for the cumulative and hierarchical nature of mathematics learning (e.g. Schmidt, 2009), and the agency of teachers to decide on the sequencing of topics within the 3 year curriculum for grades 7–9 in Sweden (Skolverket, 2018a). Previous research has collapsed the three response options to the TIMSS content coverage items to 1 =this year and 0 =before or after this year and compared these teacher reports of the implemented curriculum to national expectations at the assessed grade level to establish the alignment within each country (i.e. Burroughs et al., 2019). While the alignment of national expected teaching and realised teaching across the four cycles of TIMSS varies between counties, in the 2015 cycle of TIMSS, instruction by Swedish teachers aligned with curriculum goals 48% of the time (Burroughs et al., 2019). This lack of alignment demonstrates the

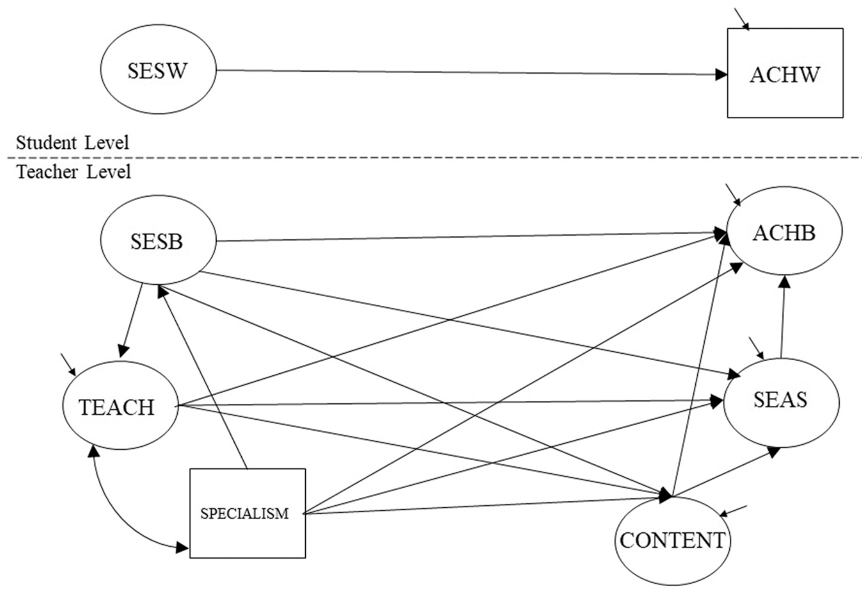


**Fig. 1.** Overall Model.

existence of teacher agency and discretion in implementing the curriculum in Swedish classrooms and lends credence to the coding regime used in these analyses. To confirm the rationale of this logic, we computed an additional content coverage measure, in which teacher responses were dichotomised, so that responses of "Not yet taught or just introduced"= 0, while "Mostly taught this year" and "Mostly taught before this year" were combined into one group with a value of 1. The same procedure of parcelling and standardization was followed, and the correlation between the original and dichotomous variables was confirmed.

### 4.4. Teacher preparedness

Teacher Preparedness was indicated by a series of items asking how prepared teachers felt to teach various mathematics topics corresponding to the content coverage items (see Foy, 2017b, p. 259) covering topics such as 'Computing with whole numbers', and 'Geometric properties of angles and geometric shapes'. The teacher responses to these items were coded 0 = 'not well prepared', 1 = 'somewhat prepared', and 2 = 'very well prepared'. Means of the items with the subdomains were taken to create four the new variables: PT: Number (5 items), PT: Algebra (6 items), PT: Geometry (6 items), and PT: Data & Chance (3 items) (for a full list of the items within each parcel, see Appendix A). Finally, these four new variable parcels, along with the teacher experience item, were standardized to a mean of 0 and a SD of 1. Descriptive statistics are shown in Table 1.

### 4.5. School emphasis on academic success

To measure School Emphasis on Academic Success (SEAS), TIMSS used 17 questions in 2015. With so many indictors, it was reasonable to assume that SEAS might be multi-factorial. As such, we conducted an Exploratory Factor Analysis (EFA) of the SEAS indictors. We specified a five-factor solution using varimax rotation. Evaluation of the EFA suggested that a four-factor solution was the most appropriate as it yielded an Eigenvalue of 1.147 and $\chi^2$ (74, 189)= 161.309, $p = .00$. The other model fit was not so promising (RMSEA=0.079 (90% CI.062–0.096), SRMR= 0.042, CFI= 0.935, TLI= 0.880), but compared to the five-factor solution, the four-factor model contained no cross-loadings, making it a preferred solution. The results of the EFA were used to create four variable parcels: SEAS: Teachers (5 items), SEAS: Parents (5 items), SEAS: Students (3 items), and SEAS: Schools (4 items) (for a full list of the items within each parcel, see Appendix A). Finally, these four new variable parcels were standardized to account for the varying number of items within each. Descriptive statistics are shown in Table 1. Parcelling the questionnaire items satisfied our theoretical understanding of SEAS as a construct driven by the behaviour of multiple actors within the school context. Again, the parcels were standardized to a mean of 0 and a SD of 1, and descriptive statistics are shown in Table 1.

### 4.6. Analysis plan

#### 4.6.1. Development of measurement models and invariance testing

This study used Mplus v8.1 (Muthén & Muthén, 1998-2017) to apply multiple-group confirmatory factor analysis (MGCFA) and measurement invariance (MI) frameworks to identify latent constructs with which to build a two-level structural equation model (Hox, Moerbeek, & van de Schoot, 2010) of the relations of teacher-level factors to mathematics achievement in Sweden. The data was grouped by the declared specialty of mathematics teachers, that is whether they are or are not mathematics specialists.

In a first analytical stage, MGCFA was used to identify a measurement model for each of the latent constructs. MGCFA is an approach to assess the psychometric qualities of a construct in two or more groups (Brown, 2015). The design of TIMSS data gathering instrument (i.e. the questionnaires) groups multiple questions relating to an overarching concept together. However, this data collection procedure alone was not enough to infer the psychometric qualities of constructs based on multiple items. As several of our latent factors had more than three indicators and would thus be over-identified, modification indices were requested to evaluate the model and find the best fitting factor structure across the two groups. The measurement model for the student level construct – SES within – utilised student responses (n = 3888), while those of the teacher level constructs (SES between, content coverage, Teacher Preparedness, and SEAS) used only teacher data (n = 190). The relatively small size of n at the teacher level was a potential source of model ill-fit (Anderson & Gerbing, 1984; Brown, 2015), and as such these factors warranted particularly close consideration.

Once appropriately fitting MGCFA models were established for each latent construct, they were tested for MI to establish their suitability for using with our two groups of teachers. The MI testing procedure tests for invariance using three models of increasing restriction in succession. The first of these, the configural model, tests for equal factor structures (i.e. the number of factors and the pattern of indicator-factor loadings).

**Table 2**
MGCFA results.

| | Non-specialist | Specialist teachers |
|---|---|---|
| Socioeconomic Status (Within) | | |
| *Loadings* | | |
| Books | .696 | .711 |
| Parental Education | .455 | .461 |
| Home possessions | .527 | .511 |
| $\chi^2$ (4, 3857)= 2.796, $p < .593$, RMSEA.000 (90% CI.000-0.029), SRMR= 0.012, CFI= 1.000 | | |
| Socioeconomic Status (Between) | | |
| *Loadings* | | |
| L2 Books | .914 | .848 |
| L2 Parental Education | .665 | .588 |
| L2 Home possessions | .793 | .797 |
| $\chi^2$ (4, 190)= 3.945, $p < .414$, RMSEA.000 (90% CI.000-0.154), SRMR= 0.048, CFI= 1.000 | | |
| Content Coverage | | |
| *Loadings* | | |
| CC: Algebra | .486 | .510 |
| CC: Geometry | .658 | .699 |
| CC: Number | .457 | .351 [ns] |
| CC: Data | .294 | .303 [ns] |
| $\chi^2$ (10, 186)= 15.742, $p < .107$, RMSEA.079 (90% CI.000-0.149), SRMR= 0.069, CFI= 0.865 | | |
| Teacher Preparedness | | |
| *Loadings* | | |
| PT: Number | .794 | .730 |
| PT: Algebra | .840 | .866 |
| PT: Geometry | .825 | .781 |
| PT: Data | .833 | .840 |
| $\chi^2$ (18, 187)= 8.068, $p < .622$, RMSEA= 0.000 (90% CI.000-0.095), SRMR= 0.050, CFI= 1.000 | | |
| SEAS | | |
| *Loadings* | | |
| SEAS: Teachers | .876 | .859 |
| SEAS: Parents | .480 | .387 |
| SEAS: Students | .490 | .405 |
| SEAS: Schools | .626 | .587 |
| *Correlations* | | |
| SEAS: Students with SEAS: Parents | .263 | .526 |
| $\chi^2$ (8, 189)= 7.280, $p < .507$, RMSEA.000 (90% CI.000-0.113), SRMR= 0.045, CFI= 1.000 | | |

Note. All loadings significant at $p < .05$, unless indicated [ns] Non-significant

Secondly, the metric model constrains factor loadings to equality. Thirdly, the scalar model holds factor intercepts to equality. Metric and scalar models allow us to compare latent correlations and latent means respectively. Accordingly, measurement models which do not meet the assumptions of invariance inherent in the configural, metric, and scalar models are unreliable foundations on which to conduct further research.

#### 4.6.2. Structural analyses of relationships

The latent factors identified in the first phase of the analysis were used to create an overall two-level structural equation model (see Fig. 1). Due to the two level nature of our data, the data was clustered by teacher, and separate weights provided in the TIMSS dataset were used for each level. The student level data was weighted by the so-called house weight, while the teacher level data was weighted by mathematics teacher weight, which accountsed for the possibility of students being taught by multiple teachers (Foy, 2017a), and the MLR estimator was used. The TIMSS dataset presents achievement as plausible values. All five plausible values were used in the structural equation modelling by utilising Mplus's imputation function.

At the student level, mathematics achievement was regressed on individual SES. At the teacher level, our area of focus, the group mathematics achievement was regressed on the four predicting factors, SES, content coverage, Teacher Preparedness, and SEAS, and the dummy variable Teacher Specialism. To assess socioeconomic inequalities in Swedish mathematics classrooms, Teacher Preparedness, content coverage, and SEAS were regressed on SES. Additional relationships between the factors were specified, as illustrated in Fig. 1. Finally a two-group structural equation model was fitted. This model followed the structure of the overall model, but used Teacher Specialism as a grouping variable and thus removed it as an indicator at the teacher level.

Throughout the analyses, a number of goodness of fit indices were evaluated. Following the recommendations of Hu and Bentler (1999) models were considered well-fitting if they were within the bounds of acceptability for both an incremental fit index (i.e. Comparative Fit indices (CFI)), and absolute fit indices (Root mean square error of approximation (RMSEA) and standardised root mean square residual (SRMR)). Models were considered to have good fit if the RMSEA < 0.06 and SRMR < 0.08. For the CFI,.95 is considered the threshold for good fit (Hu & Bentler, 1999).

## 5. Results

### 5.1. Multiple-group confirmatory factor analyses

A series of multiple-group confirmatory factor analyses were conducted to develop measurement models for the latent variables. The two SES factors on the within and between levels were just identified, containing three items each, and the model was well fitting for both groups. Teacher Preparedness, which contained four items, was also well fitting. The initial factor model for School Emphasis on Academic Success was not well fitting. However, following consultation of the modification indices a correlation between SEAS: Students and SEAS: Parents was added, which generated a good-fitting model. The measurement model for content coverage was not well fitting in terms of CFI or RMSEA. As the factor contained four items, the modification indices were consulted, but these recommended no changes to the model. As the model held acceptable RMSEA, it was retained for the next step in the analyses process. Factor loadings for the final factor models are shown in Table 2, alongside model fit information.

### 5.2. Measurement invariance

The factor structures identified in the MGCFA stage of this analysis were tested for measurement invariance using Mplus' model=configural metric scalar command, which generated three models with three levels of constraints within one model. The MLR estimator was used and the model was specified as type=complex. The model fit indices of configural, metric, and scalar MI models for each of our latent constructs are presented in Table 3.

Excepting content coverage (RMSEA=0.077, CFI=0.948), all the latent constructs fully met the criteria for configural invariance, with the RMSEA and SRMR being below.06 and.08 respectively, and CFI being greater than.95 (e.g. Hu & Bentler, 1999). With the exception of content coverage, the change in CFI and RMSEA as we progressed through the degrees of invariance was negligible and therefore scalar MI held for all factor structures. As our latent constructs achieved scalar invariance, it was reasonable to assume that they were fit for purpose in examining differing structural relations between our two groups of teachers. While content coverage did not meet the threshold for goodness of fit in terms of RMSEA or CFI in any of the three invariance models, it was determined to retain the variable for use in structural equation modelling on the basis of its acceptable SRMR values. However, the results on the model parameters that involve content coverage had to be interpreted with caution.

### 5.3. The overall model

The first structural equation model was an overall model. Fully standardised results are found in Table 4. The model was well fitting: $\chi^2$ (107, 3888)= 202.955, RMSEA= 0.015, SRMR (within)= 0.010, SRMS (between)= 0.083, CFI= 0.948, and yielded an interesting pattern of significant relationships. The crucial finding of the overall model was

**Table 3**
MI results.

| | Nº of Parameters | $\chi^2$ | df | $\chi^2_{diff}$ | Δdf | RMSEA (90% CI) | SRMR | CFI |
|---|---|---|---|---|---|---|---|---|
| Socioeconomic Status (Within) | | | | | | | | |
| Configural | 18 | .002 * | 0 | | | .000† (.000-0.000) | .000† | 1.000† |
| Metric | 16 | 1.147 | 2 | 1.145 | 2 | .000† (.000–038) | .007† | 1.000† |
| Scalar | 14 | 2.796 | 4 | 1.649 | 2 | .000† (.000-0.029) | .012† | 1.000† |
| Socioeconomic Status (Between) | | | | | | | | |
| Configural | 18 | .000 * | 0 | | | .000† (.000-0.000) | .000† | 1.000† |
| Metric | 16 | 2.481 | 2 | 2.481 | 2 | .050† (.000-0.216) | .045† | .996† |
| Scalar | 14 | 3.945 | 4 | 1.464 | 2 | .000† (.000-0.154) | .048† | 1.000† |
| Content Coverage | | | | | | | | |
| Configural | 24 | 6.203 | 4 | | | .077 (0.000-0.188) | .047† | .948 |
| Metric | 21 | 1.935 | 7 | 4.732 | 3 | .078 (0.000-0.162) | .058† | .907 |
| Scalar | 18 | 15.742 | 10 | 4.807 | 3 | .079 (0.000-0.149) | .069† | .865 |
| Teacher Preparedness | | | | | | | | |
| Configural | 24 | 2.692 | 4 | | | .000† (.000-0.130) | .017† | 1.000† |
| Metric | 21 | 6.424 | 7 | 3.732 | 3 | .000† (.000-0.121) | .049† | 1.000† |
| Scalar | 18 | 8.068 | 10 | 1.644 | 3 | .000† (.000-0.773) | .050† | 1.000† |
| School Emphasis on Academic Success | | | | | | | | |
| Configural | 26 | .336 | 2 | | | .000† (.000-0.118) | .007† | 1.000† |
| Metric | 23 | 5.375 | 5 | 5.039 | 3 | .028† (.000-0.148) | .038† | .997† |
| Scalar | 20 | 7.28 | 8 | 1.905 | 3 | .000† (.000-0.113) | .045† | 1.000† |

*p < .000; †values demonstrate acceptable fit (Hu & Bentler, 1999)

that it demonstrated a moderate inequality in access to prepared teachers (0.200) in eighth grade mathematics classrooms. The overall model did not provide evidence of significant relationships between SEAS and achievement. The model further pointed to a distinct collection of inequities, with substantial inequalities in in achievement (0.605) and content coverage (0.446). The significant correlation between Teacher Preparedness and Teacher Specialism offered a compelling reason to investigate these inequities in a two-group context to see if there was a differing pattern of relationships for specialist and non-specialist teachers.

### 5.4. The two-group model

The second structural equation model followed the specification of the overall model, but removed Teacher Specialism, which became the grouping variable. Fully standardised results are found in Table 4. The model had acceptable fit: $\chi^2$ (219, 3888)= 371.010, RMSEA= 0.019, SRMR (within)= 0.008, SRMS (between)= 0.061, CFI= 0.931. When we focused on the results at the teacher level, differing patterns of significant relationships between the latent variables emerged.

The results for the non-specialist teachers revealed a number of significant relationships. A moderate but significant inequality in access to prepared teachers was displayed in this group, with more advantaged students being taught by higher quality teachers. Additionally, there was a significant relationship between content coverage and achievement. While there was no significant inequality in content coverage (or relationship between content coverage and SES), the existence of this positive relationship between the content taught and group achievement has profound implications for the repercussions of teacher behaviours, which we explore in the discussion part of this paper. As with the overall model, there was a strong relationship between SES and achievement of.640, which indicated a substantial inequality of achievement between teaching groups of differing socioeconomic makeup. Finally, the model indicated a moderate relationship between SES and SEAS, with teachers in advantaged schools reporting a stronger academic ethos than those with less advantaged students, and Teacher Preparedness and SEAS, with more prepared teachers reporting a stronger academic ethos than their less prepared peers.

Among specialist teachers, only two of the specified relationships were significant. The model indicted a moderate inequality in content coverage between more and less socioeconomically advantaged groups of students. In classes taught by specialists, advantaged groups were exposed to more mathematics content. This inequality however, did not bear results on achievement. No predictors held significant effects on achievement, implying that the observed inequality in content coverage may have a root in pedagogic practices. The second significant relationship for the specialist teachers was that between SES and SEAS. SEAS regressed very strongly (0.805) on SES, indicating that teachers with more socioeconomically advantaged groups of students perceived their schools to have a markedly more academically orientated ethos.

## 6. Discussion and conclusions

In applying a two-level structural equation framework to TIMSS data, this study found evidence of multiple inequalities in Swedish eighth grade classrooms. In line with the earlier literature (i.e. Mullis et al., 2016; Sirin, 2005), higher SES students outperformed their lower SES peers. Additionally, as shown by earlier research (e.g. Agirdag, Van Houtte, & Van Avermaet, 2012), more socioeconomically advantaged classes scored substantially higher in mathematics. Beyond these

**Table 4**
Model results.

| | Overall model | Two-group model | |
|---|---|---|---|
| | | Non-specialist teachers | Specialist teachers |
| **Student Level** | | | |
| Socioeconomic Status by | | | |
| Books | .750 | .745 | .755 |
| Parental Education | .434 | .433 | .439 |
| Home Possessions | .491 | .498 | .482 |
| Achievement on | | | |
| Socioeconomic Status | .469 | .456 | .482 |
| **Teacher Level** | | | |
| Socioeconomic Status by | | | |
| Home Possessions | .816 | .852 | .821 |
| Books | .851 | .864 | .791 |
| Parental Education | .662 | .686 | .612 |
| Content Coverage by | | | |
| CC: Geometry | .535 | .526 | .619 |
| CC: Algebra | .442 | .440 | .51 |
| CC: Number | .449 | .460 | .401 |
| CC: Data | .463 | .376 | .441 |
| Teacher Preparedness by | | | |
| PT: Algebra | .901 | .865 | .911 |
| PT: Number | .781 | .805 | .758 |
| PT: Geometry | .811 | .811 | .815 |
| PT: Data | .857 | .840 | .860 |
| School Emphasis on Academic Success by | | | |
| SEAS: Teachers | .866 | .442 | .477 |
| SEAS: Parents | .487 | .961 | .841 |
| SEAS: Students | .388 | .752 | .654 |
| SEAS: Schools | .598 | .302 | .335 |
| SEAS: Students with SEAS: Parents | .333 | -2.095 | -0.003 $^{ns}$ |
| School Emphasis on Academic Success on | | | |
| Socioeconomic Status | .213 $^{ns}$ | .389 | .805 |
| Content Coverage | .071$^{ns}$ | -.040$^{ns}$ | -.107$^{ns}$ |
| Teacher Preparedness | .160$^{ns}$ | .264 | -0.023$^{ns}$ |
| Teacher Specialisation | .090$^{ns}$ | | |
| Content Coverage on | | | |
| Socioeconomic Status | .446 | -0.385$^{ns}$ | .448 |
| Teacher Preparedness | -0.082$^{ns}$ | -.002$^{ns}$ | -.113$^{ns}$ |
| Teacher Specialisation | .175$^{ns}$ | | |
| Teacher Preparedness on | | | |
| Socioeconomic Status | .200 | .324 | -0.020$^{ns}$ |
| Socioeconomic Status on | | | |
| Teacher Specialisation | .032$^{ns}$ | | |
| Teacher Preparedness with Teacher Specialisation | .246 | | |
| Achievement on | | | |
| Socioeconomic Status | .605 | .640 | -0.465$^{ns}$ |
| Content Coverage | .203$^{ns}$ | .367 | -0.051$^{ns}$ |
| Teacher Preparedness | -0.002$^{ns}$ | -.075$^{ns}$ | -.051$^{ns}$ |
| | -0.071$^{ns}$ | -.014 $^{ns}$ | -.138$^{ns}$ |
| School Emphasis on Academic Success | | | |
| Teacher Specialisation | .130$^{ns}$ | | |
| Model fit | $\chi^2$ (107, 3888)= 202.955, RMSEA= 0.015, SRMR (within)= 0.010, SRMS (between)= 0.083, CFI= 0.948 | $\chi^2$ (219, 3888)= 371.010, RMSEA= 0.019, SRMR (within)= 0.008, SRMS (between)= 0.061, CFI= 0.931 | |

Note. All loadings significant at $p < .05$ unless indicated $^{ns}$ non-significant
All loadings fully standardised

headline disparities, this study sought to investigate the themes of educational quality and equality in Swedish mathematics classrooms by exploring three questions: firstly, whether there is socioeconomic inequality in access to well-prepared mathematics teachers in Sweden; secondly, whether SEAS is associated with mathematics achievement; and thirdly, whether classes with non-specialist teachers experience differing patterns of inequalities when examining the relations between educational quality and educational equality factors and mathematics achievement to those with specialist teachers.

The results of our analysis provide evidence of inequalities in the distribution of prepared teachers in Sweden. Contrary to the earlier findings of Akiba et al. (2007), teacher preparedness is not distributed in a compensatory way. The overall model demonstrates that socioeconomically advantaged mathematics classrooms in Sweden are staffed with teachers who are more prepared to teach the curriculum and who have more experience. For non-specialist teachers, this inequality of teacher preparedness persists, however there is no significant inequality of teacher preparedness in specialist classrooms. This final finding is more in line with the findings of Akiba et al. (2007), and suggests that ensuring that all teachers are certified in the correct age and subject specialism, along with university coursework (as established by Rice, 2003; Wayne & Youngs, 2003) in their subject is essential for reducing inequality in provision in Swedish schools.

While the findings of this analysis provide no evidence that school culture towards achievement has a relationship with mathematics achievement in either the overall or grouped models, contrary to the model of SEAS and achievement established by Martin et al. (2013), they do highlight interesting observations in terms of the relationship between a school's culture and its community. As suggested from our review of the previous research (e.g. Harvey & Slatin, 1975; Martin et al., 2013; Pinquart & Ebeling, 2020) socioeconomic inequality in teacher perception of the school climate towards achievement is evidenced in the two group model for both specialist and non-specialist teachers, with teachers of more socioeconomically advantaged classes reporting stronger school culture towards achievement. This inequality is far more pronounced in specialist classrooms than in non-specialist ones. However, closer examination of the factor loadings within the SEAS construct indicates that parent- and student-derived items load onto the SEAS factor more strongly in the non-specialist classrooms than the specialist classrooms, and teacher- and school-derived items behave conversely. While for both groups, parental aspects of SEAS load on the factor the strongest, the.959 loading of this item in the non-specialist classes raises questions about the importance of parental expectations of success in these environments.

We observe differing patterns of inequalities for students with teachers of different specialties in Sweden. In non-specialist classrooms, there is pronounced inequality of outcomes, with advantaged classes expected to substantially outperform disadvantaged ones, while there is no significant difference in achievement in specialist classrooms.

Additionally, inequality of content coverage is not equally evidenced in the two groups of teachers, neither is inequality of access to well-prepared teachers. Non-specialist teachers are unequally dispersed among social groups, while unequal learning opportunities are provided among the specialist cohort.

An explanation for the marked differences in inequalities between the two groups of teachers might lie in the nature of the TIMSS data used in this analysis. In our sample, specialist teachers (those who majored in both mathematics and mathematics education) comprise 99 of the 190 teachers in the study, with the remaining 91 teachers classified as non-specialists. This distribution of teachers to the two groups, while statistically advantageous, seems contrary to the situation described by the Swedish Schools Agency, which has reported that approximately 85% of mathematics teachers within grades 7–9 were licenced to teach both mathematics and this age group (Skolverket, 2015). Such a discrepancy is likely due to the TIMSS sampling process which samples in two stages, first for schools according to a nationally set sampling frame, and secondly for classes within schools, and as such may draw a non-fully representative sample of teachers.

The absence of significant inequality in content coverage in cohorts with non-specialist teachers may be rooted in teacher practice. In compulsory schools, the curriculum is established in three-year blocks of content, and the current planning framework gives teachers the professional discretion to manage time across the three-year programme (Skolverket, 2018a). It is possible that specialist teachers manage their time within mathematics lessons and tailor their delivery of material to students in a different way to non-specialists, resulting in the observed inequality in content coverage between socioeconomically advantaged and disadvantaged classes.

### 6.1. Areas for further study

The confirmation of a suspected unequal access to well-prepared teachers in non-specialist taught classes, indicating that more capable non-specialist teachers are more often found to teach more affluent cohorts, raises questions surrounding the recruitment practices in Swedish schools. Swedish school principals are already required to compete for qualified teachers within the highly marketized school sector (Skolverket, 2018b). In conjunction with the inequality in learning opportunities as indicated through content coverage in specialist taught classrooms, we are minded to consider further research into the interaction between teaching practices, teacher recruitment, and student recruitment in Sweden. Furthermore, the differentiating patterns of significant relations identified in this study between the general and grouped model indicates that investigating additional teacher-derived indicators of educational quality within a model of educational equality is an important and interesting area of further study.

### 6.2. Policy implication

The findings of our analysis highlight the importance of having specialist teachers. While Sweden's National Agency for Education already reports that a high proportion of teachers in mathematics in the 7th-9th grade are qualified in both their age group and subject (per Skolverket, 2015), there exist multiple routes into teaching. Aspiring 7th-9th grade teachers may take a 4 year undergraduate programme to become a subject teacher, or those with an existing undergraduate education in their subject area can take a 1.5 year supplementary pedagogical education programme. Alternatively, experienced teachers without a degree can take courses in further education for teachers to make up sufficient credits to qualify as a teacher (Universitets- och högskolerådet, 2021). The definition used in this study of specialist teachers as having university level educations in both mathematics and mathematics education targeted the second of these three main routes into teaching, and may account for the discrepancy between the high

**Table A1**
Items in variable parcels.

| Grouped variable | Question: When have students in this class been taught each of the following mathematics topics? |
|---|---|
| CC: Number | Computing with whole numbers<br>Comparing and ordering rational numbers<br>Computing with rational numbers<br>Concepts of irrational numbers<br>Problem solving involving percents or proportions |
| CC: Algebra | Simplifying and evaluating algebraic expressions<br>Simple linear equations and inequalities<br>Simultaneous (two variables) equations<br>Numeric, algebraic, and geometric patterns or sequences<br>Representation of functions as ordered pairs, tables, graphs, words, or equations<br>Properties of functions |
| CC: Geometry | Geometric properties of angles and geometric shapes<br>Congruent figures and similar triangles<br>Relationship between three-dimensional shapes and their two-dimensional representations<br>Using appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes<br>Points on the Cartesian plane<br>Translation, reflection, and rotation |
| CC: Data & Chance | Characteristics of data sets<br>Interpreting data sets<br>Judging, predicting, and determining the chances of possible outcomes |
| **Grouped variable** | **Question: How well prepared do you feel you are to teach the following mathematics topics?** |
| PT: Number | Computing with whole numbers<br>Comparing and ordering rational numbers<br>Computing with rational numbers<br>Concepts of irrational numbers<br>Problem solving involving percents or proportions |
| PT: Algebra | Simplifying and evaluating algebraic expressions<br>Simple linear equations and inequalities<br>Simultaneous (two variables) equations<br>Numeric, algebraic, and geometric patterns or sequences<br>Representation of functions as ordered pairs, tables, graphs, words, or equations<br>Properties of functions |
| PT Geometry | Geometric properties of angles and geometric shapes<br>Congruent figures and similar triangles<br>Relationship between three-dimensional shapes and their two-dimensional representations<br>Using appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes<br>Points on the Cartesian plane<br>Translation, reflection, and rotation |
| PT: Data & Chance | Characteristics of data sets<br>Interpreting data sets<br>Judging, predicting, and determining the chances of possible outcomes |
| **Grouped variable** | **Question: How would you characterize each of the following within your school?** |
| SEAS: Teachers | Teachers' understanding of the school's curricular goals<br>Teachers' degree of success in implementing the school's curriculum<br>Teachers' expectations for student achievement<br>Teachers working together to improve student achievement<br>Teachers' ability to inspire students |
| SEAS: Parents | Parental involvement in school activities<br>Parental commitment to ensure that students are ready to learn<br>Parental expectations for student achievement<br>Parental support for student achievement<br>Parental pressure for the school to maintain high academic standards |
| SEAS: Students | Students' desire to do well in school<br>Students' ability to reach school's academic goals<br>Students' respect for classmates who excel in school |
| SEAS: School | Clarity of the school's educational objectives<br>Collaboration between school leadership and teachers to plan instruction<br>Amount of instructional support provided to teachers by school leadership<br>School leadership's support for teachers' professional development |

nationally reported proportion of qualified mathematics teachers and the even split between specialists and non-specialists in our study.

It is interesting to note that as shown in the Table 1, specialised teachers report higher levels of preparedness than non-specialist teachers do, while there is a higher variation in non-specialist teachers' responses to these items, indicating that some non-specialists feel unprepared to teach the content areas in TIMSS, while others feel very prepared. It is further possible that there is a low level of observed variation within the specialist teachers' cohort due to more uniform teaching practice which shows up as non-significant results when aggregated. Consequently, our results suggest that in these scenarios there is an unobserved pedagogic explanation for unequal opportunities. From the educational quality and equality perspective, there is motivation to explore the pedagogical implications of having multiple strands of teacher qualifications.

### *6.3. Limitations of the study*

This study explored classroom level inequalities using TIMSS data. Due to the logically small number of teachers included in TIMSS relative to the number of students, the statistical power of our analyses and our findings are potentially limited.

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### *Declaration of interest statement*

The authors report no conflict of interest in conjunction with this paper.

## Appendix A

See Table A1.

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